Given $n = \sum_{i=1}^{k} r_i$, prove that

$$\binom{n}{r_1, r_2, \ldots, r_k} = \binom{n-1}{r_1 - 1, r_2, \ldots, r_k} + \binom{n-1}{r_1, r_2 - 1, ..., r_k} + \cdots + \binom{n-1}{r_1, r_2, \ldots, r_k - 1}$$

- Using arithmetic arguments.
- Using combinatorial arguments.

The arithmetic proof is straightforward. Given the equality

$$\binom{n}{r_1, r_2, \ldots, r_k} = \frac{n!}{r_1! r_2! \cdots r_k!}$$

then we have this expansion:

$$\frac{n!}{r_1! r_2! \cdots r_k!} = \frac{(n-1)!}{(r_1 - 1)! r_2! \cdots r_k!} + \cdots + \frac{(n-1)!}{r_1! r_2! \cdots (r_k - 1)!}$$

Multiplying by $\frac{r_1! r_2! \cdots r_k!}{(n-1)!}$ gives

$$n = r_1 + r_2 + \cdots + r_k$$

which is the definition of n. Can you outline a combinatorial argument? What I want to understand is what the RHS "partitions" of the LHS symbolize.